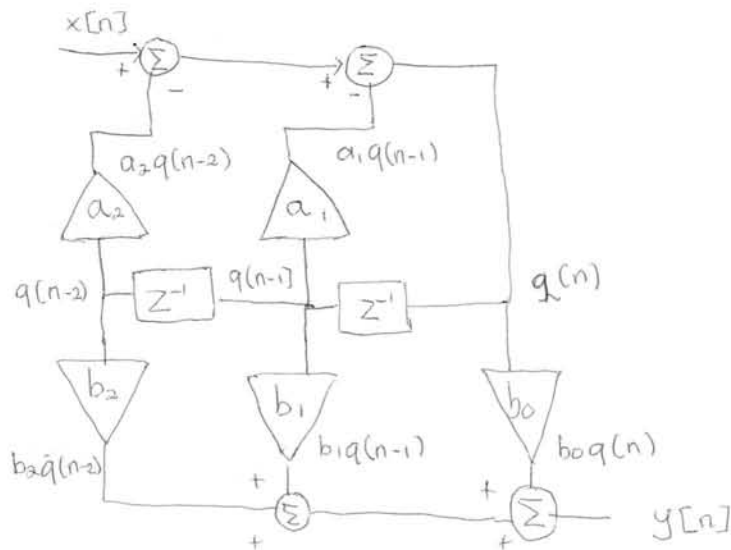


4.56. Consider the system shown in Fig. 4.10

- Find the system function $H(z)$!
- Find the difference equation relating the output $y[n]$ and input $x[n]$.



a. $H(z) = \frac{Y(z)}{X(z)}$

$$q[n] = -a_1 q[n-1] + x[n] - a_2 q[n-2]$$

$$y[n] = b_0 q[n] + b_1 q[n-1] + b_2 q[n-2]$$

→ Z transform.

$$q(z) = -a_1 z^{-1} q(z) + X(z) - a_2 z^{-2} q(z)$$

$$Y(z) = b_0 q(z) + b_1 z^{-1} q(z) + b_2 z^{-2} q(z)$$

$$\begin{aligned} X(z) &= q(z) + a_1 z^{-1} q(z) + a_2 z^{-2} q(z) \\ &= q(z) (1 + a_1 z^{-1} + a_2 z^{-2}) \end{aligned}$$

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} = \left(\frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \right) \frac{\cancel{Q(z)}}{\cancel{Q(z)}} \\ &= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \end{aligned}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y(z) = x(z) \cdot h(z)$$

$$y(z) = x(z) \left(\frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \right)$$

$$y(z) (1 + a_1 z^{-1} + a_2 z^{-2}) = x(z) (b_0 + b_1 z^{-1} + b_2 z^{-2})$$

$$y(z) + a_1 z^{-1} y(z) + a_2 z^{-2} y(z) = b_0 x(z) + b_1 z^{-1} x(z) + b_2 z^{-2} x(z)$$

$\rightarrow z^{-1}$ transform

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$