Practice Problem 10-1 — Current Mirror — ANALYSIS

Determine the following for the circuits of Figure 10-3(a) and 10-3(b) below:

a) Output current and output resistance of the current-mirror shown in Figure 10-3(a).

b) The resistance R and output resistance R_o of the current-mirror shown in Figure 10-3(b). Assume $V_A = 125$ V.

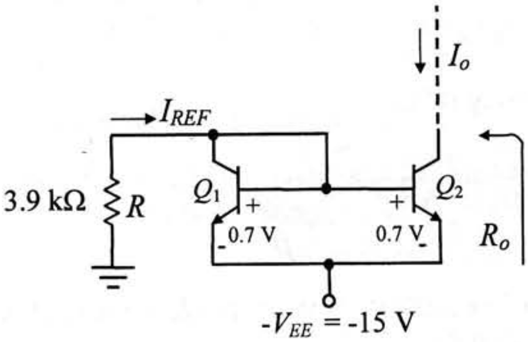


Figure 10-3(a): Basic current-mirror

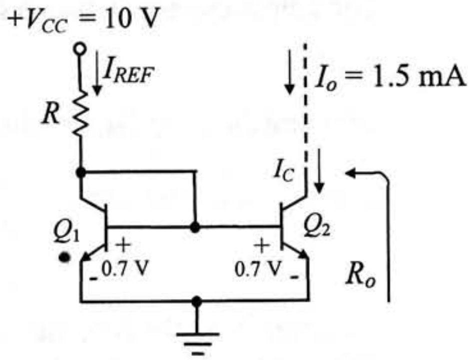
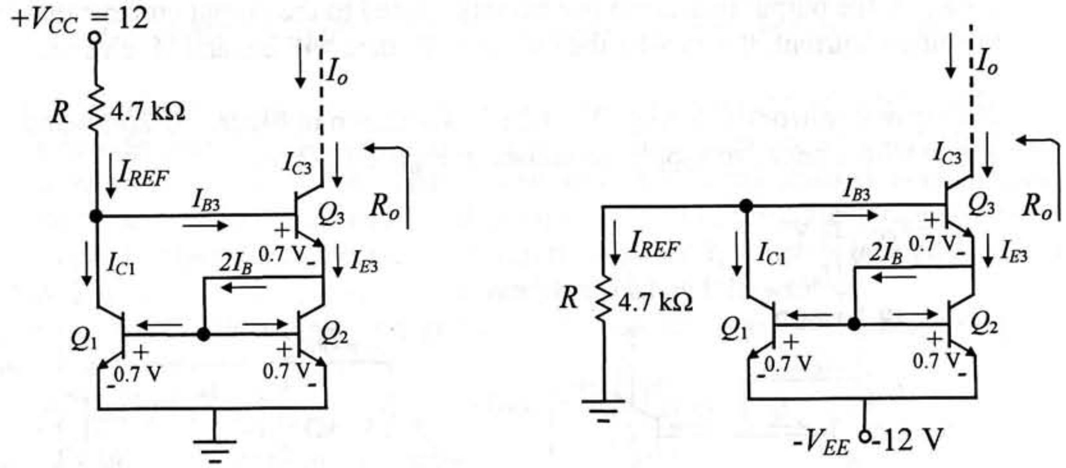


Figure 10-3(b): Basic current-mirror

Answers: a) $I_o = 3.66$ mA, $R_o = 34$ kΩ b) $R = 6.2$ kΩ, $R_o = 83.3$ kΩ

10.2.2 Wilson Current-Mirror

A circuit diagram of a Wilson current-mirror and its equivalent with negative supply is shown in Figures 10-4(a) and (b) below:



(a) Wilson *current-mirror* with positive supply

(b) Wilson *current-mirror* with negative supply

Figure 10-4: Circuit diagram of a Wilson *current-mirror*

Applying the *KVL* around the path from V_{CC} to ground results in the following:

$$V_{CC} = I_{REF} \times R + V_{BE3} + V_{BE2}$$

Rearranging the variables and solving for I_{REF}, we obtain the following:

$$I_{REF} = \frac{V_{CC} - 2V_{BE}}{R} \tag{10-4}$$

Assuming $Q_1=Q_2=Q_3$ and applying the *KCL* at the collector node of Q_2 results in the following equation:

$$I_{E3} = I_{C2} + 2I_B \tag{10-5}$$

Since Q_1 and Q_2 are biased in parallel by having their base and emitter terminals tied together, let $I_{E1} = I_{E2} = I_E$, $I_{C1} = I_{C2} = I_C$, and $I_{B1} = I_{B2} = I_B$.

Substituting for I_C *and* I_B in terms of I_E in Equation 10-5 results in the following:

$$I_{E3} = \frac{\beta}{\beta+1} I_E + 2\frac{I_E}{\beta+1} = I_E\left(\frac{\beta}{\beta+1} + \frac{2}{\beta+1}\right) = I_E\left(\frac{\beta+2}{\beta+1}\right) \tag{10-6}$$

Substituting for I_{B3} in terms of I_{E3}, we obtain the following:

$$I_{B3} = \frac{I_{E3}}{\beta+1} = I_E\left(\frac{\beta+2}{(\beta+1)^2}\right) \tag{10-7}$$

Applying the *KCL* at the collector node of Q_1 and substituting for I_C in terms of I_E results in the following equation:

$$I_{REF} = I_C + I_{B3} = \frac{\beta}{\beta+1} I_E + I_E\left(\frac{\beta+2}{(\beta+1)^2}\right) \tag{10-8}$$

Factoring out I_E and taking the common denominator, we obtain the following:

$$I_{REF} = I_E\left(\frac{\beta(\beta+1)+\beta+2}{(\beta+1)^2}\right) \tag{10-9}$$

Substituting for I_{C3} in terms of I_{E3} results in the following:

$$I_O = I_{C3} = \frac{\beta}{\beta+1} I_{E3} = \frac{\beta}{\beta+1}\left(\frac{\beta+2}{\beta+1} \cdot I_E\right) = \frac{\beta(\beta+2)}{(\beta+1)^2} I_E \tag{10-10}$$

To obtain the ratio of I_o to I_{REF}, we can simply multiply Equation 10-10 with the inverse of Equation 10-9, as follows:

$$\frac{I_o}{I_{REF}} = \frac{\beta(\beta+2)I_E}{(\beta+1)^2} \cdot \frac{(\beta+1)^2}{I_E\beta(\beta+1)+\beta+2} \tag{10-11}$$

Canceling out the common terms in the denominator and the numerator and dividing both by β results in the following:

$$\frac{I_o}{I_{REF}} = \frac{(\beta+2)}{(\beta+1)+1+\dfrac{2}{\beta}} = \frac{\beta+2}{(\beta+2)+\dfrac{2}{\beta}} \tag{10-12}$$

Dividing both the numerator and the denominator by $(\beta + 2)$ results in the following:

$$\frac{I_o}{I_{REF}} = \frac{1}{1+\dfrac{2}{\beta(\beta+2)}} = \frac{1}{1+\dfrac{2}{\beta^2+2\beta}} \tag{10-13}$$

$$\frac{I_o}{I_{REF}} \cong \frac{1}{1+\dfrac{2}{\beta^2}} \tag{10-14}$$

$$I_o \cong \frac{I_{REF}}{1+\dfrac{2}{\beta^2}} \tag{10-15}$$

For example, if $\beta = 100$, $I_o = 0.9998 I_{REF}$, and if $\beta = 200$, $I_o = 0.99995 I_{REF}$.

The output resistance R_o of the Wilson current mirror can be shown to be approximately $\frac{\beta}{2}r_o$, which is $\frac{\beta}{2}$ times higher than the output resistance of the basic current mirror.

$$R_o = \frac{\beta}{2} r_o \tag{10-16}$$

where

$$r_o = \frac{V_A}{I_{E(Q3)}} = \frac{V_A}{I_{REF}} \tag{10-17}$$

Let us now determine the bias current and the output resistance of the Wilson current mirror of Figure 10-4. Assume $V_A = 135$ V and $\beta = 200$. Then,

$$I_{REF} = \frac{V_{CC} - 2V_{BE}}{R} = \frac{(12-1.4)\text{ V}}{4.7\text{ k}\Omega} = 2.255\text{ mA} \qquad R_o = \frac{\beta}{2} r_o = 100 \times 60\text{ k}\Omega = 6\text{ M}\Omega$$

Practice Problem 10-2 Current Mirror **ANALYSIS**

Determine the reference current I_{REF}, the output current I_o, and the output resistance for the Wilson current-mirror current source of Figure 10-5.
Assume $\beta = 160$ and $V_A = 160$ V.

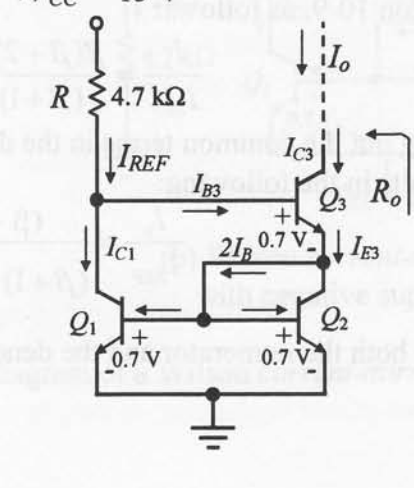


Figure 10-5: Wilson current-mirror current source

Answers: $I_{REF} = 2.89$ mA,
$I_o = 2.89$ mA, $R_o = 4.4$ MΩ

Section Summary

Current Mirror

ANALYSIS

Summary of Equations for the Analysis of Basic and Wilson Current-Mirrors

Basic current-mirror:

$$I_{REF} = \frac{V_{CC} - V_{BE}}{R}$$

$$\frac{I_o}{I_{REF}} = \frac{\beta}{\beta + 2} = \frac{1}{1 + \frac{2}{\beta}}$$

$$I_o = (1 + 2/\beta) I_{REF} \cong I_{REF}$$

$$R_o = r_o \qquad r_o = \frac{V_A}{I_o}$$

Wilson current-mirror:

$$I_{REF} = \frac{V_{CC} - 2V_{BE}}{R}$$

$$\frac{I_o}{I_{REF}} = \frac{\beta}{(\beta + 2)^2} \cong \frac{1}{1 + \frac{2}{\beta^2}}$$

$$I_o = (1 + 2/\beta^2) I_{REF} = I_{REF}$$

$$R_o = \frac{\beta}{2} r_o \qquad r_o = \frac{V_A}{I_o}$$

Practice Problem 10-3

Current Mirror

DESIGN

Design the following current sources for an output current of 4 mA, and determine its output resistance if V_{CC} = 16 V, β = 160, and V_A = 160 V.

a) Basic current-mirror current source
b) Wilson current-mirror current source

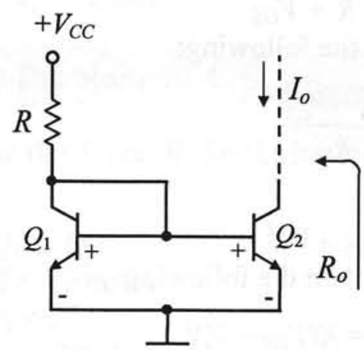


Figure 10-6(a): Basic current-mirror

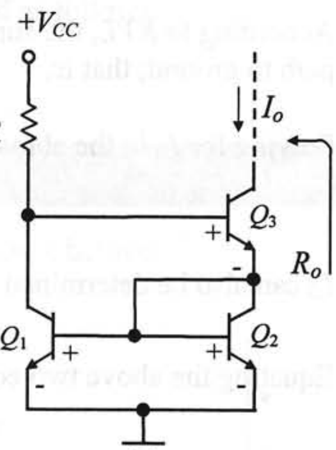


Figure 10-6(b): Wilson current-mirror

Answers: a) $R = 3.9\ \text{k}\Omega$, $R_o = 40\ \text{k}\Omega$, b) $R = 3.6\ \text{k}\Omega$, $R_o = 3.2\ \text{M}\Omega$

10.2.3 MOSFET Current-Mirror Current Sources

Basic MOSFET Current-Mirror

The basic MOSFET current-mirror can be achieved with two MOSFETs connected in a similar fashion as the basic BJT current-mirror, as shown in Figure 10-7 below:

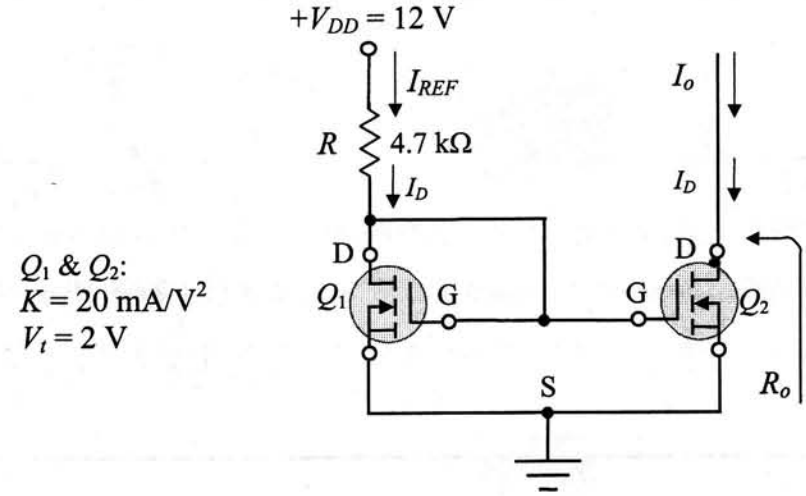


Figure 10-7: Circuit diagram of a basic MOSFET *current-mirror* current source

Since V_{GS1} equals V_{GS2}, I_{D1} equals I_{D2}, and because there is no gate current, I_{RFE} equals I_o.

$$V_{GS1} = V_{GS2} = V_{GS}$$

$$I_{D1} = I_{D2} = I_D$$

$$I_{RFE} = I_D = I_o$$

According to *KVL*, the supply voltage V_{DD} must equal the sum of voltage drops along the path to ground; that is,

$$V_{DD} = I_D \times R + V_{GS} \tag{10-18}$$

Solving for I_D in the above equation results in the following:

$$I_D = \frac{V_{DD} - V_{GS}}{R} \tag{10-19}$$

I_D can also be determined as follows:

$$I_D = K(V_{GS} - V_t)^2$$

Equating the above two equations for I_D we obtain the following:

$$I_D = \frac{V_{DD} - V_{GS}}{R} = K(V_{GS} - V_t)^2$$

$$V_{DD} - V_{GS} = KR(V_{GS} - V_t)^2$$

$$V_{DD} - V_{GS} = KR(V_{GS}^2 - 2V_{GS}V_t + V_t^2) \tag{10-20}$$

Further algebraic manipulation and separation of variables yields the following second order equation in terms of the variable V_{GS} and other known circuit parameters:

$$KRV_{GS}^2 + (1 - 2KRV_t)V_{GS} + (KRV_t^2 - V_{DD}) = 0 \tag{10-21}$$

The above equation is in the form of the following general quadratic equation:

$$ax^2 + bx + c = 0 \tag{10-22}$$

which has the following general solution:

$$V_{GS}\Big|_{n-channel} = \frac{-b+\sqrt{b^2-4ac}}{2a} \tag{10-23}$$

$$V_{GS}\Big|_{p-channel} = \frac{+b-\sqrt{b^2-4ac}}{2a} \tag{10-24}$$

where,

$$a = KR \qquad b = 1 - 2KR\,|V_t| \qquad c = KRV_t^2 - |V_{DD}|$$

After V_{GS} is determined with the above equations, I_D can be determined with Equation 10-19, but first we need to determine the parameters a, b, c, and then V_{GS} and I_D.

$a = KR = 20\text{ mA/V}^2 \times 4.7\text{ k}\Omega = 94$

$b = 1 - 2KR\,|V_t| = 1 - (2 \times 20\text{ mA/V}^2 \times 4.7\text{ k}\Omega \times 2\text{ V}) = -375$

$c = KRV_t^2 - |V_{DD}| = 20\text{ mA/V}^2 \times 4.7\text{ k}\Omega \times 4\text{ V}^2 - 12 = 364$

$$V_{GS} = \frac{+375+\sqrt{375^2-(4\times94\times364)}}{2\times94} = 2.32\text{V}$$

$$I_D = \frac{V_{DD}-V_{GS}}{R} = \frac{12\text{V}-2.32\text{V}}{4.7\text{ k}\Omega} = 2.06\text{ mA} \cong 2\text{ mA}$$

$$I_{RFE} = I_D = I_o = 2\text{ mA}$$

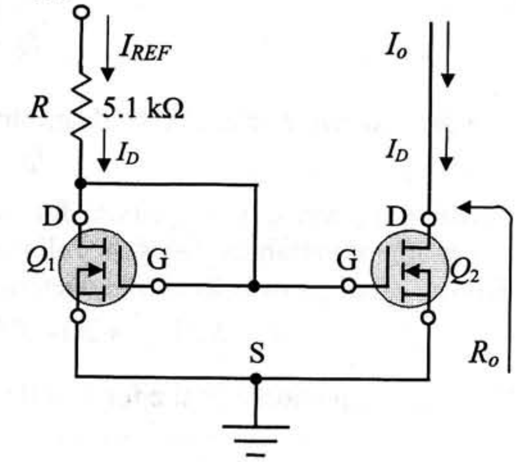
Assuming $V_A = 120$ V, the output resistance is determined as follows:

$$R_o = r_o = \frac{V_A}{I_o} = \frac{120\text{ V}}{2\text{ mA}} = 60\text{ k}\Omega$$

Practice Problem 10-4 — Current Mirror — ANALYSIS

Determine the I_o and R_o for the basic current-mirror as shown below:

Q_1 & Q_2:
$K = 20\text{ mA/V}^2$
$V_t = 2$ V
$V_A = 120$ V



Answers:

$I_o = 2.48$ mA, $R_o = 48\text{ k}\Omega$

Cascode Current-Mirror Current Source

A substantially higher output resistance can be achieved with another MOSFET current source called *cascode current mirror*, shown in Figure 10-8.

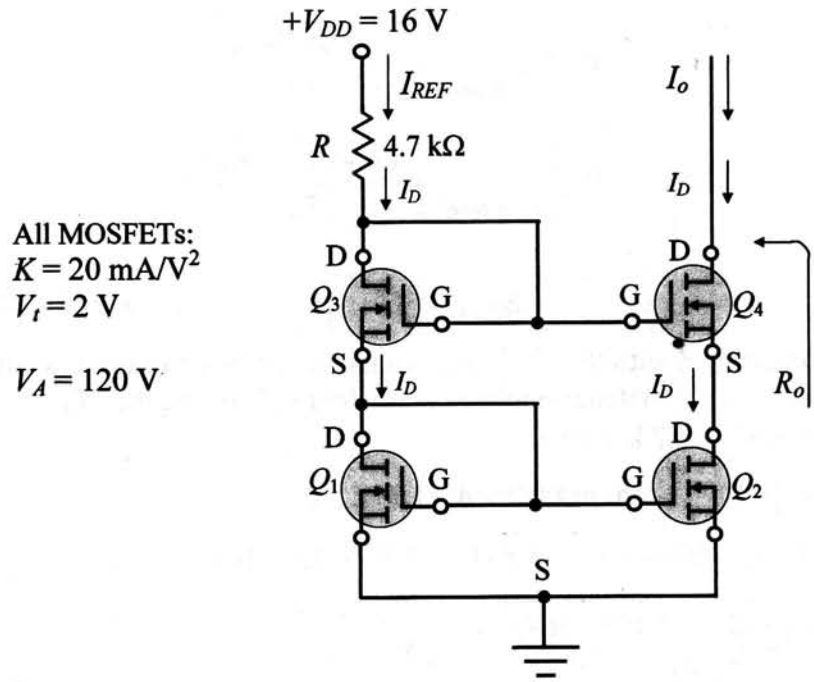


Figure 10-8: Circuit diagram of a *cascode current-mirror* current source

Each MOSFET pair in the above diagram represents a basic current mirror. Hence, all drain currents are equal to I_{REF}, and thus all gate-to-source voltages will be the same

$$V_{GS1} = V_{GS2} = V_{GS3} = V_{GS4} = V_{GS}$$
$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = I_{REF} = I_o$$

According to KVL, the supply voltage V_{DD} must equal the sum of voltage drops along the path to ground

$$V_{DD} = I_D R + 2V_{GS} \tag{10-25}$$

Solving for I_D in the above equation leads to the drain current

$$I_D = \frac{V_{DD} - 2V_{GS}}{R} \tag{10-26}$$

The dain current I_D can also be determined from Equation 7-25

$$I_D = K(V_{GS} - V_t)^2$$

Equating the above two equations for I_D and further algebraic manipulation and separation of variables yields the following second-order equation in terms of the variable V_{GS} and other known circuit parameters:

$$KRV_{GS}^{\,2} + 2(1 - KRV_t)\,V_{GS} + (KRV_t^{\,2} - V_{DD}) = 0 \tag{10-27}$$

The above equation is in the form of the following general quadratic equation:

$$ax^2 + bx + c = 0 \tag{10-28}$$

which has the following general solution:

$$V_{GS}\Big|_{n-channel} = \frac{-b+\sqrt{b^2-4ac}}{2a}, \tag{10-29}$$

where,

$$a = KR \qquad b = 2(1 - KR|V_t|) \qquad c = KRV_t^2 - |V_{DD}|$$

After the V_{GS} is determined with the above equations, I_D can be determined with Equation 10-26, but first we need to determine the parameters a, b, and c, and then V_{GS} and I_D.

$a = KR = 20\ \text{mA/V}^2 \times 4.7\ \text{k}\Omega = 94$

$b = 2(1 - KR|V_t|) = 2[1 - (20\ \text{mA/V}^2 \times 4.7\ \text{k}\Omega \times 2\ \text{V})] = -374$

$c = KRV_t^2 - |V_{DD}| = (20\ \text{mA/V}^2 \times 4.7\ \text{k}\Omega \times 4\ \text{V}^2) - 16 = 360$

$$V_{GS} = \frac{+374 + \sqrt{374^2 - (4\times 94\times 360)}}{2\times 94} = 2.35\ \text{V}$$

$$I_D = \frac{V_{DD} - 2V_{GS}}{R} = \frac{16\ \text{V} - 4.7\ \text{V}}{4.7\ \text{k}\Omega} = 2.4\ \text{mA}$$

$$I_{RFE} = I_D = I_o = 2.4\ \text{mA}$$

The output resistance of the cascode current-mirror can be shown to be as follows:

$$R_o = g_m r_o^{\,2} \tag{10-30}$$

where,

$g_m = 2K(V_{GS} - V_t) = 2\times 20\ \text{mA/V}^2(2.35\ \text{V} - 2\ \text{V}) = 14\ \text{mS}$

Assuming $V_A = 120$ V, the output resistance is determined as follows:

$$r_o = \frac{V_A}{I_o} = \frac{120\ \text{V}}{2.4\ \text{mA}} = 50\ \text{k}\Omega$$

$$R_o = g_m r_o^{\,2} = 14\ \text{mA/V} \times (50\ \text{k}\Omega)^2 = 35\ \text{M}\Omega$$

Summary of Equations for the Analysis of MOSFET Current-Mirrors

MOSFET basic current-mirror:	MOSFET cascode current-mirror:
$V_{GS}\Big\|_{n-channel} = \frac{-b+\sqrt{b^2-4ac}}{2a}$	$V_{GS}\Big\|_{n-channel} = \frac{-b+\sqrt{b^2-4ac}}{2a}$
$a = KR$, $b = 1 - 2KR\|V_t\|$	$a = KR$, $b = 2(1 - KR\|V_t\|)$
$c = KRV_t^2 - \|V_{DD}\|$, $I_{REF} = I_D = I_o$	$c = KRV_t^2 - \|V_{DD}\|$, $I_{REF} = I_D = I_o$
$I_D = \frac{V_{DD} - V_{GS}}{R}$, $R_o = r_o = \frac{V_A}{I_o}$	$I_D = \frac{V_{DD} - 2V_{GS}}{R}$, $R_o = g_m r_o^{\,2}$
	$g_m = 2K(V_{GS} - V_t)$

Practice Problem 10-5 — Current Mirror — ANALYSIS

Determine the I_o and R_o for the cascode current-mirror of Figure 10-9 below:

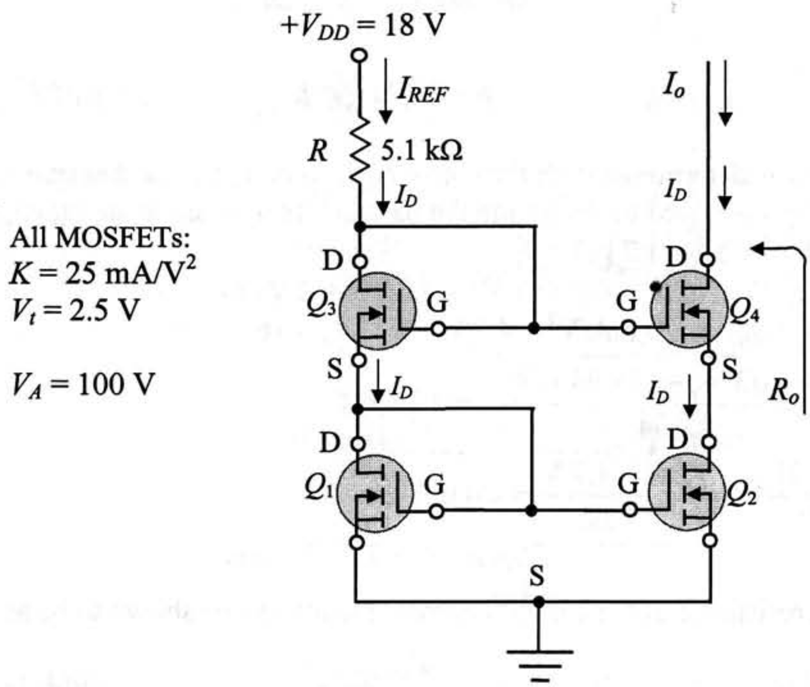


Figure 10-9: Circuit diagram of a *cascode current-mirror* current source

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Answers: $V_{GS} = 2.8115$ V, $I_D = 2.426$ mA, $R_o = 26.46$ MΩ

10.3 DIFFERENTIAL AMPLIFIER

The differential amplifier (diff-amp) is an essential component of the operational amplifier; in fact, the first two stages of an operational amplifier (op-amp) are differential amplifiers. Thus, before we begin the study of the operational amplifier and its applications, it would be useful to explore the characteristics, capabilities, and limitations of the differential amplifier. A differential amplifier is an amplifier with two input terminals and two output terminals. Its block diagram is shown below:

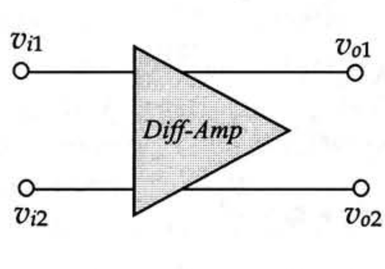

Figure 10-10: Differential amplifier block diagram

$$I_{REF} = \frac{12-0.7}{4.7\,\text{k}\Omega} = 2.4\,\text{mA}$$

To determine the relationship between the I_o and I_{REF}, we will apply *KCL* at the collector node of Q_1, as follows:

$$I_{REF} = 2I_B + I_C = 2I_B + \beta I_B = (\beta + 2)\, I_B$$

The output current I_o, which is the collector current of Q_2, is the following:

$$I_o = I_C = \beta I_B$$

Dividing the I_o by I_{REF} results in the following ratio:

$$\frac{I_o}{I_{REF}} = \frac{\beta \cdot I_B}{(\beta+2)I_B} = \frac{\beta}{\beta+2} = \frac{1}{1+\dfrac{2}{\beta}} \tag{10-2}$$

Assuming that $\beta = 200$, the above ratio will be 0.9900. That is, the I_o, which is the output current of the current source, will be the following:

$$I_o = 0.99 I_{REF}$$

$I_o = 0.99(2.4\text{ mA}) = 2.376\text{ mA}$

The output resistance of the current-mirror R_o is simply the output resistance of the transistor r_o, which can be determined with the Early voltage V_A of the transistor and the output current $I_o = I_C$ as follows:

$$R_o = r_o = \frac{V_A}{I_o} \tag{10-3}$$

Assuming $V_A = 120$ V, R_o will be approximately 50 kΩ. According to the above equation, the output resistance is inversely related to the output current; that is, the larger the output current, the smaller the output resistance will be, and vice versa.

The current-mirror of Figure 10-1, which is redrawn in Figure 10-2(a) below, may also be drawn with a negative supply, as shown in Figure 10-2(b).

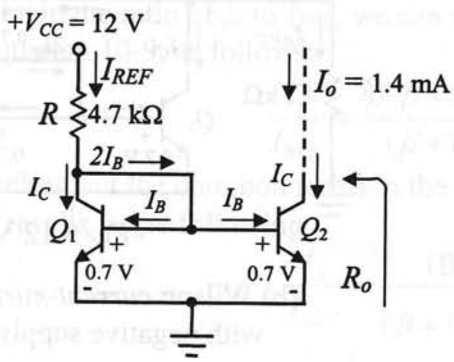

Figure 10-2(a): Circuit of a basic current-mirror with positive supply

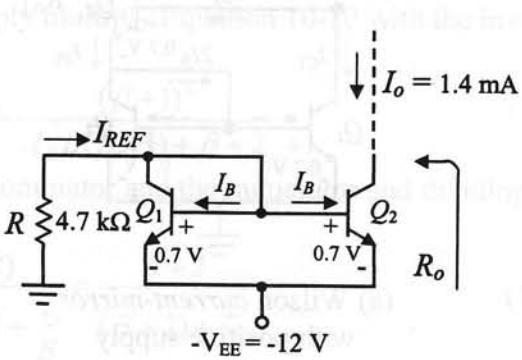

Figure 10-2(b): Circuit of a basic current-mirror with negative supply

10.2 CURRENT-MIRROR CURRENT SOURCES

An ideal current source supplies a steady amount of current and has infinite output resistance. Several types of current sources can be configured with BJTs and FETs; however, the most popular configurations that are widely used in integrated circuits are the *current-mirror* type current sources. We will discuss two types of current-mirrors, starting with the *basic* current-mirror and then, later in the chapter, the much-improved version of it, the *Wilson* current-mirror.

10.2.1 The Basic BJT Current-Mirror

As shown in Figure 10-1 below, the basic *current-mirror* current source comprises a resistor and two transistors.

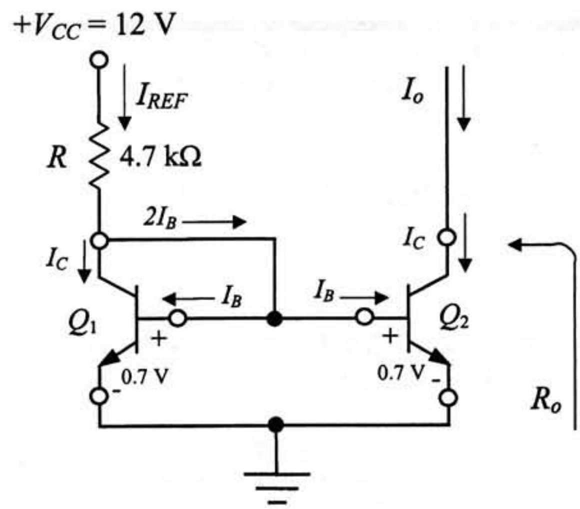

Figure 10-1: Circuit diagram of a basic *current-mirror* current source

The two transistors Q_1 and Q_2 are tied together at the base and the emitter, and both are biased through the same resistor and from the same V_{CC}. In other words, the two transistors are biased in parallel. Hence, assuming that the two transistors are identical, the base currents in both transistors will be equal, and thus, the collector currents will also be equal. The reference current through the resistor R, which is labeled I_{REF}, and its relationship to the output current I_o can be determined as follows:
According to *KVL*, the supply voltage V_{CC} must equal the sum of the voltage drops around the current path; that is,

$$V_{CC} = (I_{REF} \times R) + V_{BE}$$

Solving for I_{REF} results in the following:

$$I_{REF} = \frac{V_{CC} - V_{BE}}{R} \tag{10-1}$$